Accelerated Algebra 1 – UNIT 2
Linear and Exponential Relationships

Critical Area: Students will learn function notation and develop the concepts of domain and range. They move beyond viewing functions as processes that take inputs and yield outputs and start viewing functions as objects in their own right. They explore many examples of functions, including sequences; they interpret functions given graphically, numerically, symbolically, and verbally, translate between representations, and understand the limitations of various representations. They work with functions given by graphs and tables, keeping in mind that, depending upon the context, these representations are likely to be approximate and incomplete. Their work includes functions that can be described or approximated by formulas as well as those that cannot. When functions describe relationships between quantities arising from a context, students reason with the units in which those quantities are measured. Students explore systems of equations and inequalities, and they find and interpret their solutions. Students build on and informally extend their understanding of integer exponents to consider exponential functions. They compare and contrast linear and exponential functions, distinguishing between additive and multiplicative change. They interpret arithmetic sequences as linear functions and geometric sequences as exponential functions.

CLUSTERS	COMMON CORE STATE STANDARDS
Extend the properties of exponents to rational exponents.	**N.RN.1**. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5. **N.RN.2** Rewrite expressions involving radicals and rational exponents using the properties of exponents.
Define evaluate and compare functions **Understand the concept of a function and use function notation.**	**8.F.1.** Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. **8.F.2** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.* **8.F.3** Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.* F-IF 1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$. F-IF 2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that

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	use function notation in terms of a context. F-IF 3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n + 1) = f(n) + f(n - 1)$ for n □□1.*
Build a function that models a relationship between two quantities. *Limit to F.BF.1a, 1b, and 2 to linear and exponential functions. In F.BF.2, connect arithmetic sequences to linear functions and geometric sequences to exponential functions.*	**F.BF.1.** Write a function that describes a relationship between two quantities.★ a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.* **F.BF.2.** Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.★
Build new functions from existing functions. *Focus on vertical translations of graphs of linear and exponential functions. Relate the vertical translation of a linear function to its y-intercept. While applying other transformations to a linear graph is appropriate at this level, it may be difficult for students to identify or distinguish between the effects of the other transformations included in this standard.*	**F.BF.3.** Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*
Construct and compare linear, quadratic, and exponential models	**F.LE.1**. Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. ★ b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. ★ c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. ★ **F.LE.2**. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). ★

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Use functions to model relationships between quantities **Interpret functions that arise in applications in terms of a context.** *Focus linear and exponential functions*	8F.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. 8F.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally. **F.IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★ F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.* F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
Analyze functions using different representations. *Linear, exponential, quadratic, absolute value, step, piecewise-defined.*	**F.IF.7.** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. ★ **F.IF.9**. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.
Analyze and solve linear equations and pairs of simultaneous linear equations	8.EE.8 Analyze and solve pairs of simultaneous linear equations a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by

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	graphing the equations. Solve simple cases by inspection. *For example, 3x + 2y = 5 and 3x + 2y = 6 have no solution because 3x + 2y cannot simultaneously be 5 and 6.* c. Solve real-world and mathematical problems leading to to linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*
Solve systems of equations. *Linear-linear and linear-quadratic.*	**A.REI.5**. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions **A.REI.6**. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
Represent and solve equations and inequalities Graphically. *Linear and exponential; learn as general principle.*	**A.REI.10**. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). **A.REI.11**. Explain why the x-coordinates of the points where the graphs of the equations y=f(x) and y=g(x) intersect are the solutions of the equation f(x) = g(x) find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/ or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. ★ **A.REI.12**. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.
MATHEMATICS PRACTICES	
1. Make sense of problems and persevere in solving them. 2. **Reason abstractly and quantitatively**. 3. Construct viable arguments and critique the arguments of others. **4. Model with mathematics.** 5. Use appropriate tools strategically. 6. Attend to precision. **7. Look for and make use of structure.** 8. Look for and express regularity in repeated reasoning.	Emphasize Mathematical Practices 1, 2, 4, and 7 in this unit.

LEARNING PROGRESSIONS
CDE Progress to Algebra K-8 www.cde.ca.gov/be/cc/cd/documents/updateditem12catt3.doc Progression on HS Math - http://commoncoretools.me/wp-content/uploads/2012/12/ccss_progression_functions_2012_12_04.pdf

(m)Major Clusters – area of intensive focus where students need fluent understanding and application of the core concepts.
(s)Supporting/Additional Clusters – designed to support and strengthen areas of major emphasis/expose students to other subjects.
★Indicates a modeling standard linking mathematics to everyday life, work, and decision-making.
(+) Indicates additional mathematics to prepare students for advanced courses.

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS	KEY VOCABULARY
• Write in equivalent forms that represent both linear and exponential functions and construct functions to describe the situation and to find solutions • Apply rules that builds a function that models a relationship between two quantities • Represent equations and inequalities in one variable in various ways and use them to extend the properties of exponents to rational exponents • Understand the relationship between quantities of two systems of equations and the methods to solve two system of linear equations • Model with linear and exponential functions.	1. How will students identify the different parts of a two-system equation and explain their meaning within the context of the problem? 2. What is the importance of identifying the structure of functions and using different ways to represent them? 3. Why is it important to identify and extend the properties of exponents to rational exponents? 4. When do students decide which is the best method to solve an inequality? 5. How do you know which method to use in solving a system of equations? 6. Why is it important to analyze functions using different representations? 7. How do I analyze algebraic equations/inequalities to solve problems? 8. What must students understand in order to create equations that describe numbers or relationships? 9. How do students know which is the most efficient ways to build a function that models a relationship between two quantities? 10. Why is it important to understand solving a system of linear and exponential relationships in two variables algebraically and graphically? 11. Is there functional relationship in non-linear and ambiguous data?	Arithmetic Sequence Asymptote Boundary Coefficients Domain Exponential Explicit Function Geometric Sequence In-equalities Linear Range Rate of change Rational Recursively Symmetries

RESOURCES	INSTRUCTIONAL STRATEGIES	ASSESSMENT
Materials: **California Revised Mathematics Framework:**	***Use Analogy in the Context of the Math Exponential Growth***. When a quantity grows with	SBAC - http://www.smarterbalanced.org/

http://www.cde.ca.gov/be/cc/cd/draftmathfwchapters.asp. **Engage New York** http://www.engageny.org/sites/default/files/resource/attachments/algebra-i-m1-copy-ready-materials.pdf **Illustrative Mathematics** Skeleton Tower – F. BF.1a http://www.illustrativemathematics.org/illustrations/75 A Sum of Functions – F. BF. 1a http://www.illustrativemathematics.org/illustrations/230 Lake Algae – F. BF.1a http://www.illustrativemathematics.org/illustrations/533 Logistic Growth Model, Explicit Version: F-IF.4 http://www.illustrativemathematics.org/illustrations/804 **Inside Mathematics** http://www.insidemathematics.org/index.php/tools-for-teachers/course-1-algebra Tools for algebra **Math Assessment Project** (MAPS) Building and Solving Equations 2: A-REI http://map.mathshell.org/materials/lessons.php?taskid=554#task554 Manipulating Radicals: N-RN http://map.mathshell.org/materials/lessons.php?taskid=547#task547	time by a multiplicative factor greater than 1, it is said the quantity grows exponentially. Hence, if an initial population of bacteria, P_0, doubles each day, then after t days, the new population is given by $P(t) = P_0 2^t$ This expression can be generalized to include different growth rates, as in $(t) = P_0 r^t$. The following example illustrates the type of problem that students can face after they have worked with basic exponential functions like these. ***Example*.** On June 1, a fast growing species of algae is accidentally introduced into a lake in a city park. It starts to grow and cover the surface of the lake in such a way that the area covered by the algae doubles every day. If it continues to grow unabated, the lake will be totally covered and the fish in the lake will suffocate. At the rate it is growing, this will happen on June 30. a. When will the lake be covered halfway? b. Write an equation that represents the percentage of the surface area of the lake that is covered in algae as a function of time (in days) that passes since the algae was introduced into the lake. Facilitate a discussion that would direct students to generate recursive formula for the sequence $P(n)$, which gives the population at a given time period n in terms of the population n-1 for the following example: Populations of bacteria can double every 6 hours under ideal conditions, at least until the nutrients in its supporting culture are depleted. This means a population of 500 such bacteria would grow to 1000, etc.	PARCC - http://parcconline.org/samples/mathematics/high-school-functions F-IF.9 http://parcconline.org/sites/parcc/files/PARCC_SampleItems_Mathematics_HSAlgIMylaPool_081913_Final.pdf Myla's Swimming Pool: F-LE.2 http://parcconline.org/sites/parcc/files/HSAlg1Math2MiniGolfPrices.pdf Mini -Golf Prices: F-BF.2 http://www.ccsstoolbox.com/parcc/PARCCPrototype_main.html • Cellular growth: F-LE.2 and F-BF.2 • Rabbit populations: F-LE. 2 and 5

	Use of Exit Slips to assess student understanding. **http://daretodifferentiate.wikispaces.com/Pre-Assessment** EPR) strategies for whole group instruction. Strategies to check for understanding: Individual White Boards, Fist of Five, Exit Slip, etc.	

LANGUAGE GOALS

Students will be able to justify (orally and in writing) their rationale for solving a system of equations using various methods.

Example: To solve these equations, I use__________________________ instead of ____________________________ because______________.

Students will be able to explain (writing/speaking/listening) their understanding of the properties of the quantity represented in terms of their context.
Example: 3x – 9y = 5 and y = 1/3 x +1__________________________.

Students will be able to read a word problem and identify the language need to create an algebraic representation.

Students will be able to explain (orally and in writing) and justify their rationale for their choice of method to solve inequality equations.
Example: To solve this inequality, I use__________________________ because____________________________.

Students will be able to describe their understanding (orally and in writing) math vocabulary around whole expressions and equations.

PERFORMANCE TASKS

Illustrative Mathematics
Influenza epidemic : F.IF.4 http://www.illustrativemathematics.org/illustrations/637
Logistic Growth Model, Abstract Version : F.IF.4 http://www.illustrativemathematics.org/illustrations/800
ow is the Weather?: F.IF.4 http://www.illustrativemathematics.org/illustrations/649
Telling a Story With Graphs : F.IF.4 http://www.illustrativemathematics.org/illustrations/650

LAUSD Concept Lessons – http://math.lausd.net/middle-school/algebra-1-concept-lessons
Tying the Knots

Mathematics Assessment Project Formative Assessments/ Tasks
Comparing Investment – F.LE 1-5. http://map.mathshell.org/materials/download.php?fileid=1250
Fuctions and Everyday – F.BF.1 and F.LE.1-5 : http://map.mathshell.org/materials/download.php?fileid=1259

DIFFERENTIATION📖		
FRONT LOADING	ACCELERATION	INTERVENTION
Prerequisites • Students apply their understanding of the properties of exponents. • Students apply and extend their knowledge of rational numbers to exponents and to find the values of numerical values that include those numbers. • Students apply their knowledge about the meaning of the representation of radicals with rational exponents. • Students will understand that if the two sides of one equation are equal, and the two sides of another equation are equal, then the sum (or difference) of these is equal. • Students will extend their knowledge of learning the relationship between the algebraic representation and its graph. • Students will use their prior knowledge of creating tables of values for function to find a solution. • Students will extend their prior knowledge of graphing two equations and be able to interpret the intersection of the graph as the solution to the original equation.	• Students will design a word problem that reflects the use of graphing inequalities. • Students will write a scenario and explain the process needed to solve a system of linear equations with two variables. • Have student create a real world problem where students will build a function that model a relationship between two quantities. • Have students apply their math knowledge that will extend the properties of exponents to exponential functions. • Students will compare and contrast the properties of a linear equation and linear inequality equation.	• Use real-word context examples to demonstrate the meaning of the parts of a system of equations for the students. • Use of visual interactive websites that through the manipulation of graphs represent inequalities. • Students find it useful through technology to recognize function that represent the same relationship. • Provide a situation that uses realia to demonstrate how to build a function that model a relationship between two quantities.